## Activity description

Students use polynomial and trigonometric functions to model the number of people suffering from a cold over several weeks.

### Suitability

Level 3 (Advanced)

### Time

2–3 hours

### Resources

Student information and worksheet
Optional: slideshow

### Equipment

Computer graphing software (e.g. Excel) or graphic calculators

### Key mathematical language

Model, functions: polynomial and trigonometric.

## Notes on the activity

You could use this activity as an assignment to test what students can do unaided. You would simply give them the data on the spreadsheet and ask them to find suitable models.

Alternatively you could use this activity to teach students about how to find a model. In this case the slideshow can be used to introduce the general shape of the graph and generate discussion about possible functions.

Students are expected to use a sine function as the trigonometric model, and a quadratic function as the polynomial model. However there are other possibilities, and some students may choose to use a cosine function and/or a higher degree polynomial.

There are also a variety of ways in which the parameters can be evaluated. For example, students could substitute data pairs into a general expression then use simultaneous equations, or they could use transformations of the basic curve shapes. There will be a wide range of acceptable models.

## During the activity

Students could work individually or in pairs or small groups. They could use graph paper, graphic calculators or a spreadsheet.

## Points for discussion

At the start of the activity you could discuss the shape the data gives when plotted on a graph, and and ask students to suggest different types of function which would give approximately the same shape.

You could also ask students for ideas about how they could find the parameters.

At the end of the activity, ask students to compare and evaluate the models they have found, and consider how inaccuracies in the data might affect them.

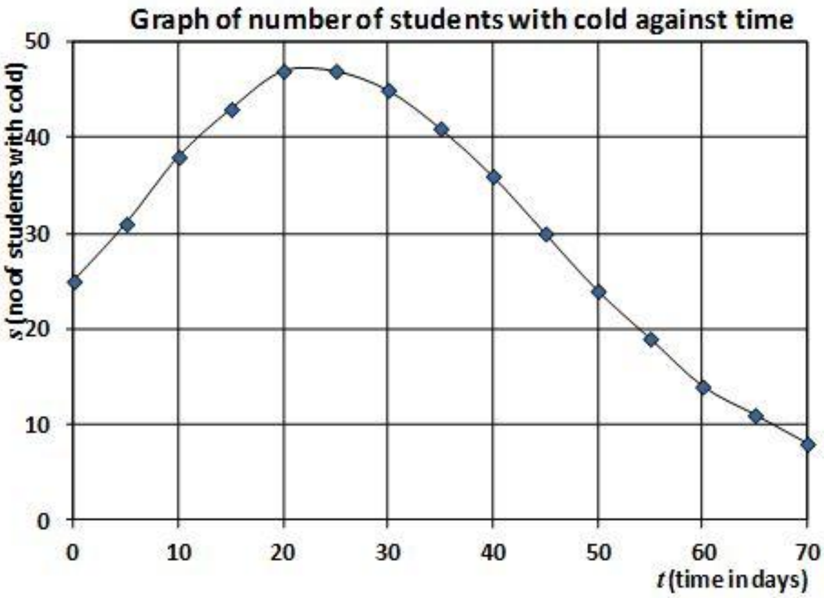
It is also worth including a discussion about the practicalities of collecting such data, and how it is unlikely that an epidemic would be detected until the number of people involved had reached a sizeable number.

## Extensions

Students could extrapolate the functions backwards to work out when the outbreak started.

## Answers

The graph below has been produced in Excel to show the given data**.**




Possible models are given below, but a wide variety of others are acceptable.

### Quadratic models

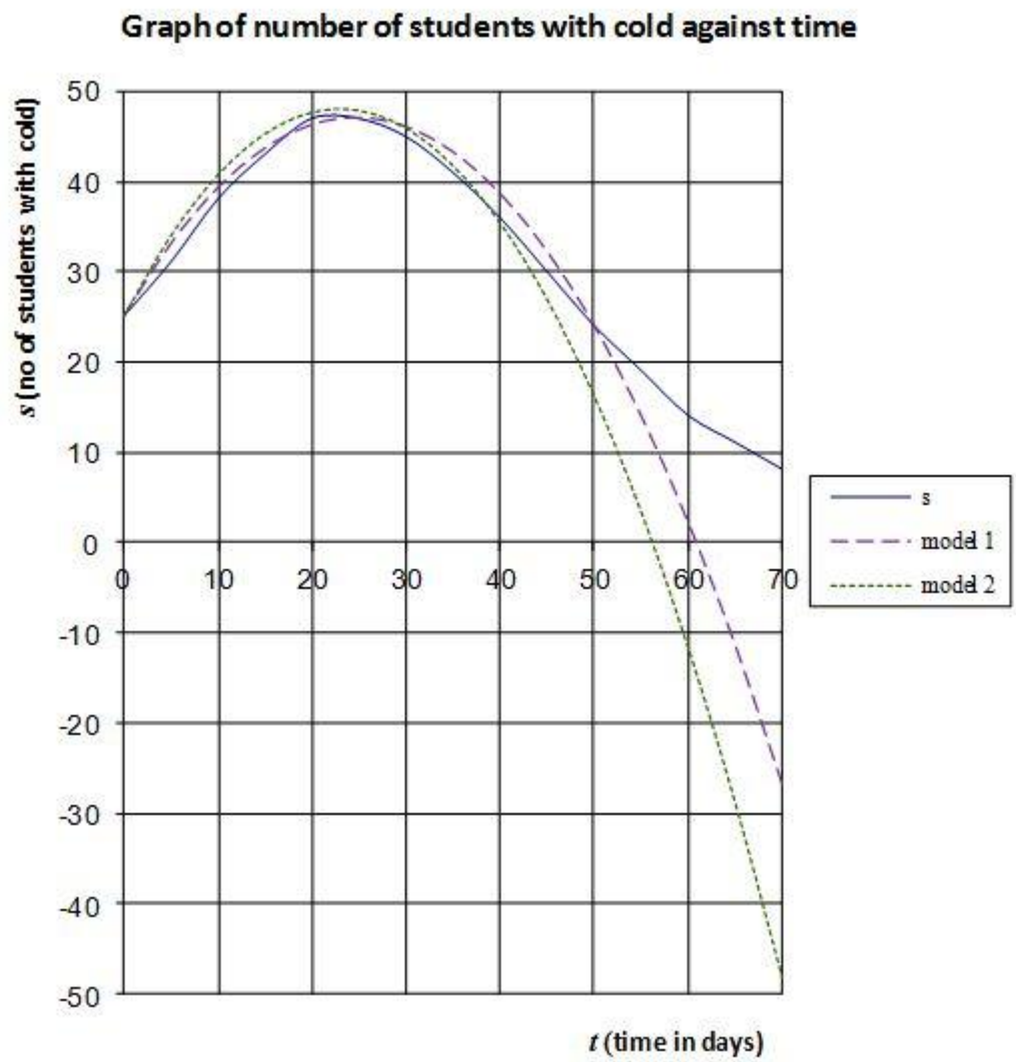
One possible method for finding a quadratic model is to substitute data pairs into the general form $s = at^2 + bt + c$ then use simultaneous equations.

For example, using the points (0, 25), (25, 47) and (50, 24) in this way gives the quadratic function $s = 25 + 1.78t - 0.036t^2$

An alternative method is to use transformations of the curve $s = t^2$ to build up a model in the form $s = a(t+b)^2 + c$. If the maximum point is taken to be (23, 48), then reflecting $s = at^2$ in the $t$ axis, translating 23 in the $t$ direction and 48 in the $s$ direction gives the graph of the function $s = 48 - a(t-23)^2$. Substituting (0, 25) then gives $a = 0.0435$ and hence the function $s = 48 - 0.0435(t-23)^2$

The graph below shows the original data and two models for $0 \le t \le 70$

Model 1: $s = 25 + 1.78t - 0.036t^2$ Model 2: $s = 48 - 0.0435(t-23)^2$




For some values of *t,* Model 1 is better and for other values, Model 2 is better. Neither function gives a good model for large values of *t*.

### Trigonometric models

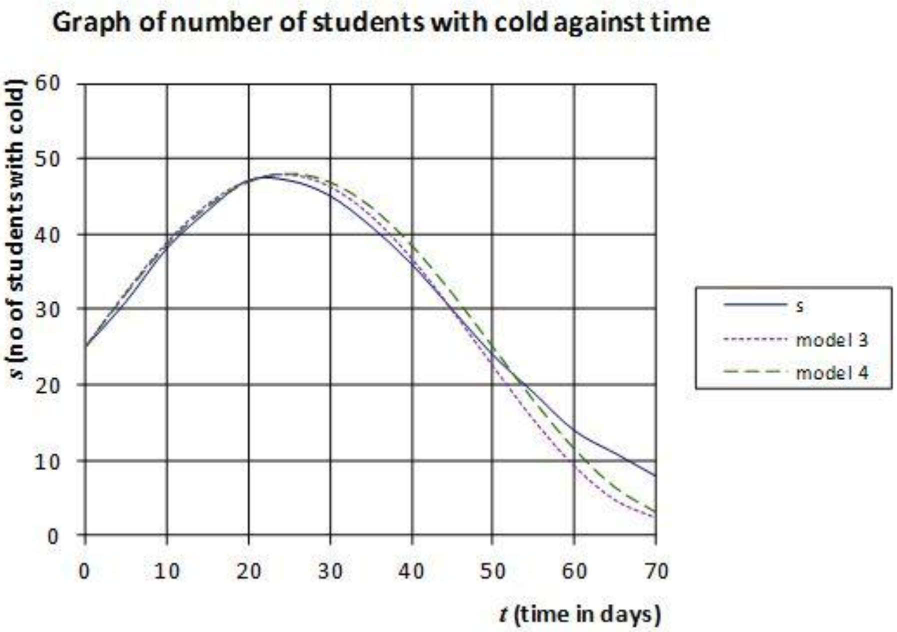
Again there are a variety of possible methods for finding a trigonometric model. Using the form $s = a\sin\omega t + c$ and taking the maximum point at (23, 48) gives the function $s = 23\sin 0.0683t + 25$. Other assumptions will lead to different trigonometric forms.

The sketch graph below shows the following models:

Model 3 $s = 23\sin 0.0683t + 25$

Model 4 $s = 23\sin 0.0633t + 25$

Both functions provide good models for $0 \le t \le 25$, but are less good for $25 \le t \le 70$. Sometimes Model 3 is the better model and sometimes Model 4.




If the more general trigonometric form $s = a\sin(\omega t + \varepsilon) + c$ is used, models can be found which follow the real data more closely for $25 \le t \le 70$.

For example, Model 5 shown below is the function
$s = 20\sin(0.0668t - 0.0668) + 27$